

## ANALYSIS OF NONLINEAR MICROWAVE CIRCUITS USING THE COMPRESSION APPROACH

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### ABSTRACT

This paper describes the Compression Approach, a technique for comprehensive analysis of nonlinear microwave circuits which supplies a framework for combining linear field simulation methods and harmonic balance. The foundations of this approach are discussed. Simulation results of this method for a nonlinear transmission line are compared with those of a pure time-domain approach.

### INTRODUCTION

During the last years there has been much progress in nonlinear circuit simulation techniques [1, 2, 3] as well as in the capabilities of numerical full-wave analysis of circuit components or even complete small-sized circuits [4, 5], partly supported by continuously increasing available computer power.

In particular, time-domain methods are increasingly used for the simulation of circuits including field-theoretical aspects as well as the behaviour of nonlinear devices by including nonlinear device models into the time-domain simulation [6, 7]. This allows for an accurate simulation of the circuit behaviour if the circuit is subject to a specific excitation. Unfortunately there are kinds of analysis which are hardly supported by the pure time-domain approach or require an enormous amount of computation time; this includes noise analysis, stability analysis, multiple analysis for different parameter sets or the analysis of mixer-like cases, i.e. under excitations which contain a number of tones of incommensurable frequencies or signals made up of signal components having large time-scale differences.

The techniques primarily dedicated to the analysis of nonlinear microwave circuits, namely frequency-domain methods of the harmonic balance (HB) type or Volterra-series analysis, have reached a high state of maturity with respect to several of these kinds of analysis [8].

Usually these methods require the division of the total circuit into "a linear subcircuit" and "a nonlinear subcircuit." In practice the description of the linear part is in most cases accomplished by performing a conventional linear analysis of some equivalent circuit which is made up of single components like lines, discontinuity models etc.,

many of these components being described by scattering parameters which are measured or derived from theoretical considerations. However, for a scattering parameter based interface between segments it is mandatory that each two ports which are joined refer to essentially the same mode spectrum in the common reference plane. The segmentation procedure will cause accuracy problems if the linear equivalent circuit does not match this condition either because parasitic effects (resulting from improper interconnection of components) are not included sufficiently, or because of the impossibility to locate proper reference planes for parts of the circuit at all.

The latter may especially be the case when nonlinear elements are located in inner parts of complex passive structures, which makes it impossible to establish reference planes in which a well defined mode spectrum suited for scattering parameter determination may be found. Circuits containing LUFET's [9] give an example for this kind of situation.

### THE COMPRESSION APPROACH

Pure time-domain methods that perform full-wave analysis are not subject to these problems because the circuit is simulated (ideally) as a whole, without the need for an artificial introduction of segments or reference planes. The compression approach is intended to combine the capabilities of field-simulation methods which are able to comprehensively account for field-theoretical properties, with the powerful analysis possibilities of frequency domain nonlinear circuit simulators [10, 11].

The compression approach views the circuit as a superposition of a single linear space domain described by Maxwell's equations, and a number of embedded nonlinear elements which are considered to be lumped. The entire passive linear space domain, including the regions where the physical nonlinear devices are located, is described ideally by a single matrix  $S_c$  of formal scattering parameters (in this context we will call this matrix the "compression matrix") plus appropriate reference resistances. "Inner" ports (see below) serve as an interface between the embedding linear space domain and nonlinear elements; the nonlinear elements are assumed to be connected to these inner ports. The nonlinear elements shall be described by standard CAD models.

The equivalent circuit used in the compression approach,

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made up of the compression matrix with nonlinear elements connected to inner ports and sources/loads/other parts of the circuit connected to outer ports, may then be analysed with a HB simulator.

The demand for a single compression matrix which describes the entire linear space domain may be weakened in case that the circuit may safely be cut into pieces which are connected by, say, lines on which an undisturbed mode spectrum can be found. Individual compression matrices will then be used to describe each linear part with respect to both inner and "outer" ports, which may be introduced on the connecting lines.

### INNER PORTS

An essential feature of the method is the concept of inner ports, i.e. ports which are located within complex metalisation structures, being connected to nonlinear elements but not to any reference line.

The standard use of scattering parameters requires assumptions on field distributions in reference planes which have a significant spatial extent. However, this is not a proper interface to nonlinear elements which are considered to be lumped. Therefore, from a geometrical point of view, an inner port is given as a path which is to be used as current path as well as for voltage integration. A nonlinear element connected to that port is thought of as being limited in its spatial extent to that path. Because inner ports may be considered to be lumped ports as opposed to the distributed ports which are needed for standard segmentation, there is no need to consider any mode spectra at inner ports. While reference planes may be thought of as defining the physical borders of a circuit component where other components may be attached, thus implying constraints on the field distribution, an inner port is completely embedded in a component, devoted exclusively to the connection of a lumped nonlinear element. However, it is convenient still to use scattering parameters for characterization purposes, but because their meaning has changed these are merely formal scattering parameters at inner ports.

The voltage of an inner port is defined to be the integral of electric field strength along the path. It is assumed that path length is short enough that no wave effects occur along the path. Current is taken to be the closed-curve integral of magnetic field strength along a curve which closely surrounds the current path.

Each port is member of a "local group," where all ports of a local group are located in such a close distance that Kirchhoff's laws can be assumed to be applicable within that group. Most groups will contain only one port. All ports of a given local group share a virtual ground. Voltage between ports of different groups is undefined because in general no electric potential may be defined.

A consequence is that connections between ports of different groups (e.g. virtually by means of the circuit simulation software) are not permissible and will yield meaningless results. Components which are both nonlinear and distributed have to be "broken" into lumped nonlinear el-

ements, each in its own local group (e.g. FET's with long gate width may be broken into several "slices"). Ports belonging to different groups are connected exclusively by means of the compression matrix.

Nonlinear CAD models which relate voltages and currents and the embedding linear space described in terms of formal scattering parameters are interfaced by

$$(\mathbf{I} - \mathbf{S}'_c) \mathbf{v} - (\mathbf{I} + \mathbf{S}'_c) \mathbf{Z} \mathbf{i} = 0, \quad (1)$$

where  $\mathbf{S}'_c = (s'_{ij})$ ,  $s'_{ij} = s_{ij} \sqrt{Z_i/Z_j}$ ,  $\mathbf{Z} = \text{diag}(Z_i)$ .  $s_{ij}$ ,  $Z_i$ ,  $\mathbf{v}$  and  $\mathbf{i}$  are scattering parameters, reference resistances and vector of port voltages and currents, respectively.

The elements of the compression matrix are to be determined from port voltages and currents. In this case Eq.(1) exactly reproduces the relation of these quantities for any reference resistance. It is important to note that for "non-physical" reference resistances all corresponding scattering parameters must not be interpreted as to relate physical wave quantities. At inner ports, where there are no physical meaningful reference resistances available, there is actually no need to consider wave quantities at all, because Eq.(1) contains the complete information required to interface nonlinear models. Therefore, at inner ports, the reference resistances may be interpreted as being temporary scaling factors which may in principle be chosen arbitrarily.

### COMPRESSION APPROACH VS. TIME DOMAIN ANALYSIS

Analysis of linear microwave circuit components is usually aimed at determining scattering parameters in a frequency range of interest. A comprehensive full-wave analysis of a particular structure may require a substantial amount of computation time, which, however, in turn yields a characterization of the structure. Including even a single nonlinear element in a full-wave time-domain analysis changes the situation significantly, because further on every change in excitation requires a completely new analysis, with almost all of the computation time devoted to the re-simulation of linear parts which in principle may be characterized in a few runs. The advantage of full-wave time-domain analysis including nonlinear elements compared to a segmentation of the circuit combined with HB is that nearly arbitrary locations for nonlinear devices are permissible. To preserve this property for HB analysis, the standard reference plane based characterization of the linear part which is used in segmentation has to be modified. Inner ports are introduced to address this item, providing an interface tailored to the connection of lumped nonlinear elements.

It has to be stressed that the determination of a compression matrix may be accomplished by any full-wave method; obviously, there is no particular restriction to time domain methods as FD-TD or TLM. In fact, frequency domain methods like e.g. the spectral domain approach (SDA) or FD-FD are suited as well. Moreover, if the frequencies to be considered in the nonlinear analysis are known beforehand, these may be picked for the linear analysis to save computation time.

## RESULTS

As an example, a nonlinear transmission line (NLTL) has been analysed with the compression approach as well as by a pure time-domain full-wave analysis approach [5] including lumped nonlinear diode models. The structure is shown in Fig. 1.

The results that are achieved with the compression approach represent solutions of the same equations that are evaluated by time-domain integrating methods. Therefore, results of both methods should be identical within numerical precision. However, it is important to note that there are fundamental differences in the way how the results are achieved. Frequency domain methods are capable of determining quasi-periodic solutions that represent steady-states, whereas time-domain integrating methods are in principle not subject to restrictions with respect to signal waveforms (if stability criterion and sampling theorem are obeyed). In order to compare results of both methods, it is indispensable that time-domain results have reached a steady state. In the context of microwave circuits, mostly steady-state solutions are of interest; therefore the time that an integrating method needs to achieve these is a measure for the practical applicability of that method in CAD environments.

A comparison of the time-domain waveform results of both methods is shown in Fig. 4 (one period of the excitation frequency). There are slight differences between both results visible indicating that transients have still not completely vanished.

Once the compression matrix has been determined, it is possible to investigate the influence of changes of the active devices' parameters with the same accuracy as with a pure time-domain analysis, but with much less computational effort. As an example, Fig. 2 shows how the signal waveform at port 18 of the NLTL depends on an additional diode series resistance  $R_s$  and the built-in voltage  $V_B$  of the diode capacitance, respectively.

The effort required for a pure time-domain approach is approximately proportional to the number of performed analyses (for this example app. 2 CPU-hours per analysis), because for each analysis the complete linear space domain has to be re-simulated over and over again. On the other hand, the compression approach required for the above example an initial effort of app. 10 CPU-hours, which was devoted to the characterization of the linear part of the NLTL. After the linear part had been characterized, each of the subsequent harmonic balance simulations took less than 3 minutes, except the very first one which was started from scratch without any solution estimate as starting value (Fig. 3). It can be seen that the compression approach yields a considerable saving in computer time, if more than a "break-even-number" of in this case six analyses have to be performed (Fig. 2 shows the result of 21 and 17 analyses, resp.).

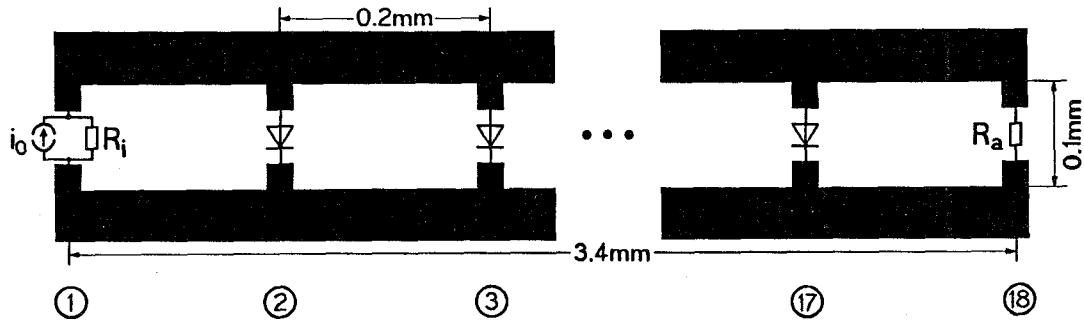
## CONCLUSION

A technique that allows for including an accurate de-

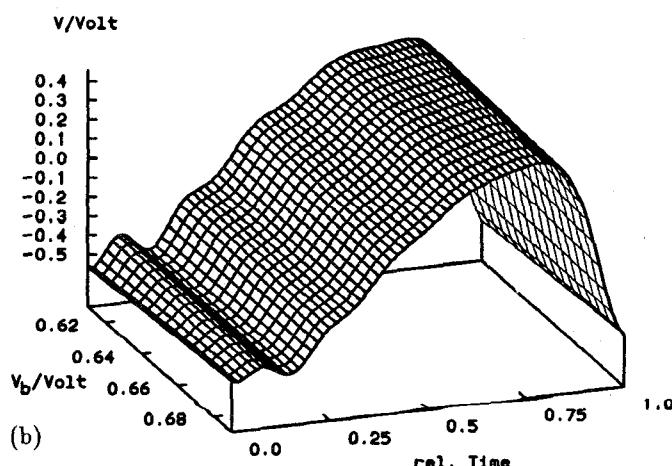
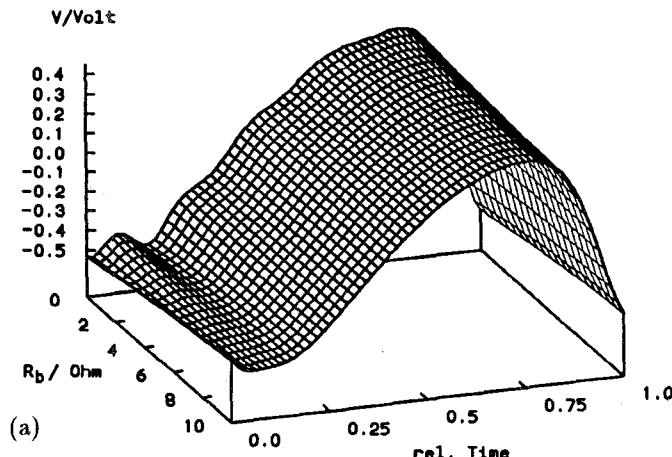
scription of linear embedding space into HB simulations of nonlinear microwave circuits has been discussed. A comparison with a pure time-domain approach has been made for the analysis of a nonlinear transmission line. The comparison shows that the presented method yields a significant saving in computer time if more than a few analyses have to be performed, while preserving the accuracy of the pure time-domain analysis.

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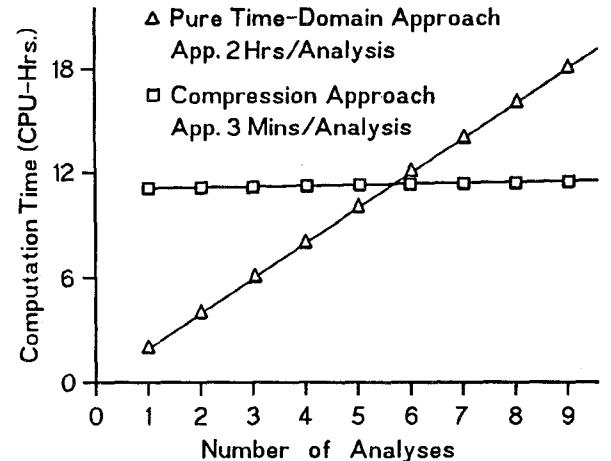
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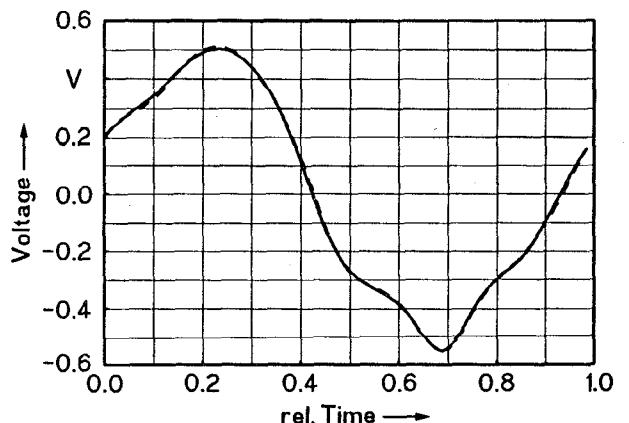
**Fig. 1:** Nonlinear transmission line (after [7], modified). Length:  $N = 16$  diodes, diode spacing =  $200\mu\text{m}$ , total length =  $3.4\text{ mm}$ . Diode:  $i(v) = I_0(\exp(v/\eta V_T) - 1)$ ,  $c(v) = C_{j0}/(1 - v/V_b)^\nu$ , with  $I_0 = 18.4\text{pA}$ ,  $\eta V_T = 38.775\text{mV}$ ,  $C_{j0} = 62.6\text{fF}$ ,  $V_b = 0.65\text{V}$ ,  $\nu = 0.5$ . Line: Coplanar strips on GaAs,  $w = 50\mu\text{m}$ ,  $s = 100\mu\text{m}$ . Excitation:  $i_0 = 20\text{mA}@10\text{GHz}$ ,  $R_i = R_a = 50\Omega$ .



**Fig. 2:** Voltage at port 18 of the NLTL as function of  
 (a) the diode series resistance  $R_b$ .  
 (b) the diode capacitance built-in voltage  $V_b$ .



**Fig. 3:** Computation times for the NLTL.



**Fig. 4:** Voltage at port 7 of the NLTL.  
 - - - FD-TD ending at 9630 timesteps.  
 —— Compression approach.